RB-0763

Second Year B. Sc. (Computer Science) Examination

April / May – 2010

Mathematics : Paper - IV

(Old Course)

Time : 3 Hours] [Total Marks : **105**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **S.Y. B.Sc. (COMPUTER SCIENCE)**

Name of the Subject :
☛ **MATHEMATICS - 4 (OLD)**

☛ Subject Code No. : **0 7 6 3** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

1 Answer the following questions : **15**

(1) Show that the relation R on the XY-plane defined by $(x, y) R (s, t)$ whenever $x - s$ or $y - t$ is an integer, is not transitive.

(2) Define :
– Partially order set
– Totally ordered set
– Well - ordered set.

(3) Show that in a lattice, if $a \le b \le c$ then

$$(a * b) \oplus (a * c) = (a \oplus b) * (a \oplus c)$$

(4) Draw a simple graph with 5 vertices and 2 components.

(5) Show that the number of leaves on an m-ary tree with i interior vertices is $(m-1)i+1$

2 (a) Prove that the relations R on the xy-plane defined by $(x, y)R(s, t)$ whenever $s-x$ and $t-y$ are both integers, defines an equivalence relation. **18**

(b) What is transitive closure of a relation ? Prove that it is transitive.

(c) Define the matrix of a relation.

$$\text{If } R_M = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} \begin{array}{c} \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

then find R_M, R_M^2 and the matrix of relation for R_M^2.

OR

2 (a) Define a congruence relation. Prove that relation modulo 5 is an equivalence relation. **18**

(b) Define the composition of a relation. Let $A = \{a, b, c, d, e\}$ and let $R = \{(a, b), (a, a), (a, c), (c, c), (b, d), (b, b), (d, d)\}$. Find the ordered pairs in the composition of a relation with itself. Draw a diagraph for R and R^2.

(c) Find R^T (transitive closure of R) for the following relation :

$A = \{a, b, c, d, e\}$ and

$R = \{(a, b), (b, c), (c, d), (d, a), (a, e)\}$

3 (a) Let A be a lattice under the relation R and let a and b be elements of A. Prove that **18**

$a \wedge (a \vee b) = a$ and $a \vee (a \wedge b) = a$.

(b) Show that D_{24} (positive divisors of 24) is a poset. Draw its Hasse diagram.

(c) Find the glb and lub of the following subsets :

(i) $S = \{x | x \in R, 1 \le x < 2\}$ where $S \subset R$ under "less than or equal to" relation.

(ii) $S = \{1, 2, 3, 4, 5\}$ where $S \subset N$ undeer "less than or equal to" relation.

OR

3 (a) Prove that every lattice is a totally ordered set. Give an example showing that a totally ordered set may not be a lattice. **18**

(b) Let $A = \{a, b, c\}$ and $P(A)$ be the power set of A. Show that $P(A)$ is a lattice under the relation "is subset of". Also draw its Hasse diagram.

(c) Prove that every chain is a distributive lattice.

4 (a) Prove that for any $a, b \in L$, $a \le b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$ **18**

(b) Show that the lattice $\langle S_n, D \rangle$ for $n = 216$ is isomorphic to the direct product of lattices for $n = 8$ and $n = 27$

(c) Give the Karnaugh map representation for the following functions :

(i) $f = \overline{x}\,\overline{y}\,z + \overline{x}\,y\,\overline{z} + x\,y\,\overline{z}$

(ii) $g = x\,\overline{y}\,\overline{z} + x\,\overline{y}\,z + \overline{x}\,\overline{y}\,z + \overline{x}\,y\,\overline{z}$

OR

4 (a) Let $\langle L, \le \rangle$ be a lattice. Prove that for any $a, b, c \in L$, **18**

$$b \le c \Leftrightarrow \begin{cases} a * b \le a * c \\ a \oplus b \le a \oplus c \end{cases}$$

(b) Define a distributive lattice. Give one example of a lattice which is distributive and also one example which is not.

(c) Obtain the product of sums canonical forms of the Boolean expressions :

(i) $x_1 * x_2$

(ii) $x_1 \oplus x_2$

5 (a) Explain the three utilities problem. **18**

(b) State and prove the necessary and sufficient condition for any graph to be disconnected.

(c) Justify - Every complete graph is a regular graph. Give an example showing that a regular graph may not be complete.

OR

5 (a) Define Euler line and Euler circuit. State and prove the necessary and sufficient condition for a graph to be an Euler graph. **18**

(b) Show that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

(c) Draw a graph with 17 edges such that 3 of its vertices have degree 3 and the rest have degree 5, if possible.

6 (a) State and prove Euler's formula for planar graphs. **18**

(b) Prove that an m-ary tree of height h has atmost m^h leaves. Find the height of a balanced ternary tree with 3001 vertices.

(c) Define a spanning tree. Draw all such trees for k_3.

OR

6 (a) If an m-ary tree is balanced and has q leaves then prove that $m^{h-1} < q \le m^h$ **18**

(b) Determine the number of regions defined by a connected planar graph with 4 vertices and 8 edges. Draw such a graph.

(c) What is the height of a balanced ternary tree with 73 vertices ?